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Annihilation, glueball and flavor mixing effects in pseudoscalar mesons

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Abstract. The pseudoscalar mesons $\eta(547)$, $\eta'(958)$ and $\eta''(1410)$ are studied in the gluonium-quarkonium mixing framework. The SU(3)-flavor symmetry breaking and annihilation effects are considered. Estimates of the glueball mass and of the m_s/m_u ratio are provided. The system $\eta(1295)$ and $\eta(1490)$ is also considered in a mixing scheme.

1 Introduction

The SU(3)-flavor symmetry breaking yields the Gell-Mann-Okubo mass formula [1,2] and the physical states η and η' are considered as mixtures of the isoscalar singlet and octet. However, these states do not satisfy a well-balanced mass relation and this fact indicates that other effects should be included. The quark-antiquark annihilation into gluons affects only the self-conjugate mesons and has been considered by some authors in attempting to solve the η-η' mass splitting. A SU(3)-invariant annihilation amplitude leads to a mass formula that fails to fit the pseudoscalar masses [3]. De Rujula et al. [4] suggested that the quark-antiquark annihilation mechanism might not be SU(3)-invariant.

In a previous paper [5] the η-η' mass splitting is explained in SU(3)-flavor symmetry breaking framework. The physical states are mixtures of the isoscalar singlet and octet states and the amplitudes of quark-antiquark annihilation into gluons are supposed to be flavor dependent. Within this formulation an extended expression for the Schwinger sum rule is satisfied. Also the mixing angle obtained, $\theta = -19.51^o$, is consistent with the experimental data ($\theta \simeq -20^o$) from η and η' decays into pions [6]. The model works well, but the result gives a hint that some significant effect possibly has not been considered. The strange result is that the ratio $m_s/m_u \simeq 2$ took a somewhat large value, in comparison with that one used in the constituent quark models ($m_s/m_u \simeq 1.3 - 1.8$).

The η-η' mixing scheme could be enlarged by the existence of glueballs. The $\eta(1440)$ was interpreted as a strong glueball candidate due its enhanced production in a gluon-rich channel [7,8]. The $\eta(1440)$, with the same quantum numbers as the η and η' system, motivated the study of the η-η'-$\eta(1440)$ mixing arrangement [9–14].

Recently, the mass region near to $\eta(1440)$ has been resolved into the two states $\eta''(1410)$ and $\eta(1490)$ [15–17]. The first one has been interpreted as being mainly a glueball mixed with $q\bar{q}$ and the second one as mainly a $s\bar{s}$ radially excited state [18,19]. Therefore one is attempted to identify $\eta''(1410)$ as the remaining physical state in this extended mixing scheme for ground states [18–20]. On the other hand, the state $\eta(1490)$ is interpreted as a partner of the radially excited state $\eta(1295)$ [19]. The states $\eta(1295)$ and $\eta(1490)$ are the physical manifestations of a 2S excited states mixing including solely light and strange quarks [18].

In this paper we describe the η-η'-η'' and $\eta(1295)$-$\eta(1490)$ systems with the same formalism used in [5]. The small overlapping of the respective mass intervals suggests the possibility of mixing among ground states and radial excitations as considered by [21], however, in a first approximation, we assume that this 1S-2S mixing may be neglected. In our approach the binding energies, differing from all the other formalisms, are considered as being flavor dependent. The same occurs with the annihilation amplitudes responsible for the $q\bar{q} \leftrightarrow gg \leftrightarrow q'\bar{q}'$ and $q\bar{q} \leftrightarrow gg$ transitions. All the relevant quantities are determined by the eigenvalues and eigenvectors of the mass matrix. This formalism has five parameters but only two of them are free, one is the glueball mass and the other one is a parameter related to the different binding energies. In searching for the best results of the branching ratios and of the decay widths involving the η, η' and η'' mesons we have fixed all the parameters of the problem. As in our previous paper [5] the ratio m_s/m_u is obtained as a by-product instead of an input as it is usual. We will see this enlarged mixing arrangement furnishes satisfactory results for the experimental data and remedies the high value for the ratio m_s/m_u obtained in [5]. Finally we extend the mixing

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$$M = \begin{pmatrix} 2m_u + E_{uu} + A_{uu} & A_{ud} & A_{us} & A_{ug} \\ A_{du} & 2m_d + E_{dd} + A_{dd} & A_{ds} & A_{dg} \\ A_{su} & A_{sd} & 2m_s + E_{ss} + A_{ss} & A_{sg} \\ A_{gu} & A_{gd} & A_{gs} & 2m_g + E_{gg} + A_{gg} \end{pmatrix} \quad (1)$$

scheme for the excited states using the value of m_s/m_u determined for the ground state.

2 Quarkonium-gluonium mixing

To enlarge the $q\bar{q}$ mixing we add a two gluon state to the flavor basis. We assume that the states of the basis are $|u\bar{u}>$, $|d\bar{d}>$, $|s\bar{s}>$ and $|gg>$. The mass matrix in this basis reads: (see (1) on top of the page.) The contribution to the elements of the mass matrix are: The rest masses of the quarks and the gluon, the eigenvalue E_{ij} of the Hamiltonian for the stationary bound state (ij) and the amplitudes A_{ij}, where $(i, j = u, d, s, g)$ that account for the possibility of $q\bar{q} \leftrightarrow gg \leftrightarrow q'\bar{q}'$ and $q\bar{q} \leftrightarrow gg$ transitions. As in the previous paper we assume that E_{ij} and A_{ij} are not SU(3)-invariant quantities [5].

The mass matrix can be conveniently rewritten in the basis of the states with $I_3 = 0$ as follows:

$$|a> \equiv |\tilde{\pi}^0> = \frac{1}{\sqrt{2}}\left(|u\bar{u}> - |d\bar{d}>\right) \quad (2)$$

$$|b> \equiv |\eta_8> = \frac{1}{\sqrt{6}}\left(|u\bar{u}> + |d\bar{d}> - 2|s\bar{s}>\right) \quad (3)$$

$$|c> \equiv |\eta_1> = \frac{1}{\sqrt{3}}\left(|u\bar{u}> + |d\bar{d}> + |s\bar{s}>\right) \quad (4)$$

$$|G> \equiv |gg> \quad (5)$$

Where $|\eta_8>$ and $|\eta_1>$ are the SU(3) isoscalar octet and singlet, respectively, $|\tilde{\pi}^0>$ is an isovector state and $|G>$ a glueball state. We use the mass relations

$$m_{\pi^+} = m_u + m_d + E_{ud} \quad (6)$$

$$m_{K^+} = m_u + m_s + E_{us} \quad (7)$$

$$m_{K^0} = m_d + m_s + E_{ds} \quad (8)$$

where the annihilation effects are absent from these non self-conjugate mesons, only the rest masses of the quarks and the binding energies contribute to their physical masses. Defining

$$\varepsilon_1 \equiv \frac{1}{2}(E_{uu} + E_{dd}) - E_{ud} \quad (9)$$

$$\varepsilon_2 \equiv \frac{1}{2}(E_{uu} + E_{ss}) - E_{us} \quad (10)$$

$$\varepsilon_3 \equiv \frac{1}{2}(E_{dd} + E_{ss}) - E_{ds} \quad (11)$$

the elements of the symmetric mass matrix in the basis $|a>$, $|b>$, $|c>$, $|G>$ read

$$M_{aa} = m_{\pi^+} + \varepsilon_1 + A_{aa} \quad (12)$$

$$M_{ab} = \frac{1}{\sqrt{3}}(m_{K^+} - m_{K^0} + \varepsilon_2 - \varepsilon_3) + A_{ab} \quad (13)$$

$$M_{ac} = \sqrt{\frac{2}{3}}(m_{K^+} - m_{K^0} + \varepsilon_2 - \varepsilon_3) + A_{ac} \quad (14)$$

$$M_{ag} = A_{ag} \quad (15)$$

$$M_{bb} = \frac{1}{3}(2m_{K^+} + 2m_{K^0} - m_{\pi^+} + 2\varepsilon_2 + 2\varepsilon_3 - \varepsilon_1) + A_{bb} \quad (16)$$

$$M_{bc} = \frac{\sqrt{2}}{3}(2m_{\pi^+} - m_{K^+} - m_{K^0} + 2\varepsilon_1 - \varepsilon_2 - \varepsilon_3) + A_{bc} \quad (17)$$

$$M_{bg} = A_{bg} \quad (18)$$

$$M_{cc} = \frac{1}{3}(m_{K^+} + m_{K^0} + m_{\pi^+} + \varepsilon_2 + \varepsilon_3 + \varepsilon_1) + A_{cc} \quad (19)$$

$$M_{cg} = A_{cg} \quad (20)$$

$$M_{gg} = M_G \quad (21)$$

where $M_G = 2m_g + E_{gg} + A_{gg}$ is the mass of the glueball state.

We adopt an expression for the amplitude of the process $q\bar{q} \leftrightarrow gg \leftrightarrow q'\bar{q}'$ similar to that of Cohen and Lipkin [22] and Isgur [23], where the numerator of the two-gluon annihilation amplitude expression is assumed to be a SU(3) invariant parameter, which means that we parameterize the annihilation amplitude in the form

$$A_{qq'} = \frac{\Lambda}{m_q m_{q'}} \quad (22)$$

Analogously the amplitude for the processes $q\bar{q} \leftrightarrow gg$ is parameterized by

$$A_{qg} = \frac{\Lambda_g}{\sqrt{m_q}} \quad (23)$$

according to the results of Close et al. [19] and Kühn et al. [24]. The phenomenological parameters Λ and Λ_g are to be determined. With these parameterizations, the amplitudes appearing in the elements of mass matrix in the basis $|a>$, $|b>$, $|c>$, $|G>$ are:

$$A_{aa} = \frac{1}{2}\left(1 - \frac{m_u}{m_d}\right)^2 \frac{\Lambda}{m_u^2} \quad (24)$$

$$A_{ab} = \frac{1}{2\sqrt{3}}\left(1 - \frac{m_u}{m_d}\right)\left(1 + \frac{m_u}{m_d} - 2\frac{m_u}{m_s}\right)\frac{\Lambda}{m_u^2} \quad (25)$$

$$A_{ac} = \frac{1}{\sqrt{6}}\left(1 - \frac{m_u}{m_d}\right)\left(1 + \frac{m_u}{m_d} + \frac{m_u}{m_s}\right)\frac{\Lambda}{m_u^2} \quad (26)$$

$$A_{ag} = \frac{1}{\sqrt{2}}\left(1-\sqrt{\frac{m_u}{m_d}}\right)\frac{\Lambda_g}{\sqrt{m_u}} \quad (27)$$

$$A_{bb} = \frac{1}{6}\left(1+\frac{m_u}{m_d}-2\frac{m_u}{m_s}\right)^2\frac{\Lambda}{m_u^2} \quad (28)$$

$$A_{bc} = \frac{1}{3\sqrt{2}}\left[\left(1+\frac{m_u}{m_d}\right)^2 -\frac{m_u}{m_s}\left(1+\frac{m_u}{m_d}+2\frac{m_u}{m_s}\right)\right]\frac{\Lambda}{m_u^2} \quad (29)$$

$$A_{bg} = \frac{1}{\sqrt{6}}\left(1+\sqrt{\frac{m_u}{m_d}}-2\sqrt{\frac{m_u}{m_s}}\right)\frac{\Lambda_g}{\sqrt{m_u}} \quad (30)$$

$$A_{cc} = \frac{1}{3}\left(1+\frac{m_u}{m_d}+\frac{m_u}{m_s}\right)^2\frac{\Lambda}{m_u^2} \quad (31)$$

$$A_{cg} = \frac{1}{\sqrt{3}}\left(1+\sqrt{\frac{m_u}{m_d}}+\sqrt{\frac{m_u}{m_s}}\right)\frac{\Lambda_g}{\sqrt{m_u}} \quad (32)$$

The eigenvalues of the mass matrix are the physical masses of the pseudoscalar mesons $\pi^0(140)$, $\eta(547)$, $\eta'(958)$ and $\eta''(1410)$. Henceforth we assume the SU(2)-flavor invariance which implies that $m_u = m_d$. This assumption is justified by a previous work in which we have shown that the SU(2)-flavor symmetry breaking is important to the mass splitting between the π^o and $\pi^{\pm}$, but it has negligible effects in the η-η' mixing [25].

Assuming the exact isospin symmetry we obtain $\varepsilon_1 = 0$, $\varepsilon_2 = \varepsilon_3$ and $A_{aa} = A_{ab} = A_{ac} = A_{ag} = 0$. This implies that $M_{aa} = m_{\pi^0} = m_{\pi^+}$ and $M_{ab} = M_{ac} = M_{ag} = 0$. Thus the mass matrix decouples and henceforth we will work only in the isoscalar subspace (I=0) generated by $|b>, |c>, |G>$.

The invariants of the mass matrix give the following mass relations:

$$m_\eta + m_{\eta'} + m_{\eta''} = tr(M) \quad (33)$$

$$m_\eta . m_{\eta'} . m_{\eta''} = det(M) \quad (34)$$

$$m_{\eta''}.m_\eta + m_{\eta''}.m_{\eta'} + m_{\eta'}.m_\eta = \frac{1}{2}\left[(tr(M))^2 - tr\left(M^2\right)\right] \quad (35)$$

The eigenvectors of the mass matrix are the physical particles η, η' and η'' which are mixtures of G, η_8 and η_1.

$$|\eta> = -s_2\,|G> + c_1c_2\,|\eta_8> - c_2s_1\,|\eta_1> \quad (36)$$

$$|\eta'> = -c_2s_3\,|G> + (c_3s_1 - c_1s_2s_3)\,|\eta_8> + (c_1c_3 + s_1s_2s_3)\,|\eta_1> \quad (37)$$

$$|\eta''> = c_2c_3\,|G> + (c_1s_2c_3 + s_1s_3)\,|\eta_8> + (c_1s_3 - s_1s_2c_3)\,|\eta_1> \quad (38)$$

The coefficients appearing in the eigenvectors were written in terms of three Euler angles defining a rotation in a three dimensional space. For brevity, we have defined the notation $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$ $(i = 1, 2, 3)$.

The eigenvectors (36)-(38) can also be rewritten in the basis $|q\bar{q}> \equiv \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}>$, $|s\bar{s}>$ and $|G>$:

$$|\eta> = X_\eta|q\bar{q}> + Y_\eta|s\bar{s}> + Z_\eta|G> \quad (39)$$

$$|\eta'> = X_{\eta'}|q\bar{q}> + Y_{\eta'}|s\bar{s}> + Z_{\eta'}|G> \quad (40)$$

$$|\eta''> = X_{\eta''}|q\bar{q}> + Y_{\eta''}|s\bar{s}> + Z_{\eta''}|G> \quad (41)$$

The invariants of the mass matrix are functions of m_s/m_u, Λ/m_u^2, $\Lambda_g/\sqrt{m_u}$, ε_2 and M_G. These quantities are not all free. The equations (33)–(35) impose some constraints among them. These equations were solved for m_s/m_u, Λ/m_u^2 and $\Lambda_g/\sqrt{m_u}$ which are functions of ε_2 and M_G. Fixing the values of ε_2 and M_G, the independent parameters of the model, all the remaining quantities become determined. The value of M_G was limited to the interval between the masses of the pseudoscalar mesons η' and η'', in order to keep the mass matrix Hermitian, because outside this interval Λ_g becomes a complex number. For a given value of M_G the parameter ε_2 is determined by the minimum of m_s/m_u, consistent with the usual values in the nonrelativistic constituent quark models, which are in the range 1.3–1.8 GeV. Now M_G is the remaining free parameter. For the determination of M_G we search for the best values for the data from the branching ratios and from the decay widths as described below.

The branching ratios and electromagnetic decay widths are given by the expressions [10,20,27,28]:

$$\mathrm{BR}(\phi \to \eta'\gamma) = \left[\frac{m_\phi^2 - m_{\eta'}^2}{m_\phi^2 - m_\eta^2}\right]^3\left(\frac{Y_{\eta'}}{Y_\eta}\right)\mathrm{BR}(\phi \to \eta\gamma) \quad (42)$$

$$\Gamma(\rho \to \eta\gamma) = \left[\frac{(m_\rho^2 - m_\eta^2)m_\omega}{(m_\omega^2 - m_\phi^2)m_\rho}\right]^3 X_\eta^2\,\Gamma(\omega \to \phi^0\gamma) \quad (43)$$

$$\Gamma(\phi \to \eta\gamma) = \left[\frac{(m_\phi^2 - m_\eta^2)m_\omega}{(m_\omega^2 - m_\phi^2)m_\phi}\right]^3 \frac{4}{9}\,Y_\eta^2\,\frac{m_s}{m_u}^2\;\Gamma(\omega \to \phi^0\gamma) \quad (44)$$

$$\Gamma(\eta' \to \rho\gamma) = 3\left[\frac{(m_{\eta'}^2 - m_\rho^2)m_\omega}{(m_\omega^2 - m_\phi^2)m_{\eta'}}\right]^3 X_{\eta'}^2\,\Gamma(\omega \to \phi^0\gamma) \quad (45)$$

$$\mathrm{BR}(D_s^+ \to \eta'\phi^+) = \left[\frac{\left(m_{D_s^+}^2 - (m_{\eta'} + m_\phi)^2\right)\left(m_{D_s^+}^2 - (m_{\eta'} - m_\phi)^2\right)}{\left(m_{D_s^+}^2 - (m_\eta + m_\phi)^2\right)\left(m_{D_s^+}^2 - (m_\eta - m_\phi)^2\right)}\right]^{1/2} \times\left(\frac{Y_{\eta'}}{Y_\eta}\right)\mathrm{BR}(\mathrm{D_s^+} \to \eta\phi^+) \quad (46)$$

$$\mathrm{BR}(J/\psi \to \omega\eta) = X_\eta^2\left|\frac{k_{\eta\omega}}{k_{\rho\phi}}\right|^3\mathrm{BR}(J/\psi \to \rho^0\phi^0) \quad (47)$$

$$\mathrm{BR}(J/\psi \to \omega\eta') = X_{\eta'}^2\left|\frac{k_{\eta'\omega}}{k_{\rho\phi}}\right|^3\mathrm{BR}(J/\psi \to \rho^0\phi^0) \quad (48)$$

$$\frac{\mathrm{BR}(J/\psi \to \phi\eta)}{\mathrm{BR}(J/\psi \to \phi\eta')} = \left(\frac{Y_\eta}{Y_{\eta'}}\right)^2\left|\frac{k_{\eta\phi}}{k_{\eta'\phi}}\right|^3 \quad (49)$$

$$\mathrm{BR}(J/\psi \to \rho^0\eta) = X_\eta^2\left|\frac{k_{\eta\rho}}{k_{\omega\phi}}\right|^3\mathrm{BR}(J/\psi \to \omega\phi^0) \quad (50)$$

$$\mathrm{BR}(J/\psi \to \rho^0\eta') = X_\eta^2\left|\frac{k_{\eta'\rho}}{k_{\omega\phi}}\right|^3\mathrm{BR}(J/\psi \to \omega\phi^0) \quad (51)$$

$$\mathrm{BR}(J/\psi \to \omega\eta'') = \left(\frac{X_{\eta''}}{X_\eta}^2\right) \left|\frac{k_{\eta''\omega}}{k_{\eta\omega}}\right|^3 \mathrm{BR}(J/\psi \to \eta\omega) \quad (52)$$

$$\mathrm{BR}(J/\psi \to \rho^0\eta'') = \left(\frac{X_{\eta''}}{X_\eta}^2\right) \left|\frac{k_{\eta''\rho}}{k_{\eta\rho}}\right|^3 \mathrm{BR}(J/\psi \to \eta\rho^0) \quad (53)$$

$$\mathrm{BR}(J/\psi \to \phi\eta'') = \left(\frac{Y_{\eta''}}{Y_\eta}^2\right) \left|\frac{k_{\eta''\phi}}{k_{\eta\phi}}\right|^3 \mathrm{BR}(J/\psi \to \phi\eta) \quad (54)$$

$$\mathrm{BR}(J/\psi \to \phi\eta'') = \left(\frac{Y_{\eta''}}{Y_{\eta'}}^2\right) \left|\frac{k_{\eta''\phi}}{k_{\eta'\phi}}\right|^3 \mathrm{BR}(J/\psi \to \phi\eta') \quad (55)$$

The mass of the glueball state given by our fit to the branching ratios and decay widths is

$$M_G = 1.300 \text{ GeV}. \quad (56)$$

The values of the remaining parameters are:

$$\varepsilon_2 = -0.004 \text{ GeV} \quad (57)$$

$$\frac{m_s}{m_u} = 1.772 \quad (58)$$

$$\frac{\Lambda}{m_u^2} = 0.301 \text{ GeV} \quad (59)$$

$$\frac{\Lambda_g}{\sqrt{m_u}} = 0.130 \text{ GeV}. \quad (60)$$

The eigenvectors obtained are:

$$|\eta> = -0.128|G> + 0.930|\eta_8> + 0.346|\eta_1> \quad (61)$$

$$|\eta'> = -0.463|G> - 0.0364|\eta_8> + 0.808|\eta_1> \quad (62)$$

$$|\eta''> = 0.877|G> - 0.057|\eta_8> + 0.476|\eta_1> . \quad (63)$$

Comparing these numerical coefficients with the angular relations of (36), (37) and (38) we obtain the mixing angles $\theta_1 = -20.4^o$, $\theta_2 = 7.34^o$ and $\theta_3 = 27.8^o$. The angle θ_1 is the mixing angle of the states $|\eta_1>$ and $|\eta_8>$ in the physical states $|\eta>$ and $|\eta'>$. This value is to be compared with the one usually adopted which is $\theta_1 = \theta_p \simeq -20^o$ [6]. A recent determination of the η_8-η_1 mixing angle in η and η' assuming corrections due to non-ideal ω-ϕ mixing [26] gives the value $-16.9^0 \pm 1.7^0$. The gluonic content of the pseudoscalar mesons given by (61), (62) and (63) are 1.64%, 21.4% and 76.9% for η, η', η'' respectively.

The eigenvectors coefficients in the basis $|q\bar{q}>$, $|s\bar{s}>$ and $|G>$ are

$$X_\eta = 0.819, \; Y_\eta = -0.559, \; Z_\eta = -0.128 \quad (64)$$

$$X_{\eta'} = 0.450, \; Y_{\eta'} = 0.764, \; Z_{\eta'} = -0.463 \quad (65)$$

$$X_{\eta''} = 0.356, \; Y_{\eta''} = 0.321, \; Z_{\eta''} = 0.877 \quad (66)$$

The value for the pseudoscalar glueball mass (56) is to be compared with those predicted by other η-η'-η'' mixing schemes: 1.369 GeV [18] and 1.302 GeV [20]. It must be observed that the mass of the pseudoscalar glueball given by our model, similarly to some other mixing schemes, is lower than the mass obtained in lattice calculations [29]. In fact there is an incompatibility between these approaches. Contrarily to what is obtained in lattice results in the quenched approximation, in the mixing schemes the pseudoscalar glueball is not assumed to be an isolated physical state. The mass of the glueball state is obtained simultaneously with the masses of the $q\bar{q}$ and $s\bar{s}$ pseudoscalar states that are also components of the physical states. This is probably the source of the considerable difference between the mass estimatives given by these approaches.

Our results for the branching ratios and decay widths are shown in Table 1 and are to be compared with those from the model of [20] and with experimental data. It must be emphasized that in our model the mixing angles which appear in the eigenvectors (36)-(38) depend only on the glueball mass.

3 Mixing in excited states

The radially excited 2S states $\eta(1295)$ and $\eta(1490)$ also may be described in the two gluon annihilation and flavor mixing framework. The absence of a third I=0 pseudoscalar 2S state gives a hint that there are no gluonium contribution to the masses of these two mesons. We take this suggestion as an assumption and use the same procedure given in [5] for the ground states η and η'. The computations need not to be reproduced here because the only difference from [5] is that all the magnitudes correspond to excited states.

The masses of the physical mesons which appear in [5] become now: $m_\eta = 1.295$ GeV, $m_{\eta'} = 1.490$ GeV, $m_\pi = 1.300$ GeV and $m_K = 1.460$ GeV. The parameters obtained for the excited states, maintaining the ratio $m_s/m_u = 1.772$ determined in the Sect. 2 for the η-η'-η'' mixing scheme, are

$$\varepsilon^* = -0.002 \text{ GeV} \quad (67)$$

$$\frac{\Lambda^*}{m_u^2} = -0.065 \text{ GeV} \quad (68)$$

where the asterisk refers to the excited states.

The states $\eta(1295)$ and $\eta(1490)$ are mixtures of the excited octet and singlet:

$$|\eta(1295)> = \cos\theta^* \; |8>^* - \sin\theta^* \; |1>^* \quad (69)$$

$$|\eta(1490)> = \sin\theta^* \; |8>^* + \cos\theta^* \; |1>^* \quad (70)$$

where the mixing angle is $\theta^* = 55.3^0$. If these states are written in the basis $|q\bar{q}>^* \equiv \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}>^*$ and $|s\bar{s}>^*$ we obtain

$$|\eta(1295)> = 0.999|q\bar{q}>^* + 0.010|s\bar{s}>^* \quad (71)$$

$$|\eta(1490)> = -0.010|q\bar{q}>^* + 0.999 \; |s\bar{s}>^* . \quad (72)$$

Table 1. Branching ratios and electromagnetic decay widths involving the η, η' and η''. Our results are compared with other model and with the experimental data

Observable	Our Model	Model [20]	Experiment [6]
$\mathrm{BR}(\phi \to \eta'\gamma)$	1.05×10^{-5}	$(5.6 \pm 0.3) \times 10^{-5}$	$< 4.1 \times 10^{-4}$
$\Gamma(\rho \to \eta\gamma)$	6.1×10^{-2} MeV	$(5.0 \pm 0.3) \times 10^{-2}$ MeV	$(5.7 \pm 1.4) \times 10^{-2}$ MeV
$\Gamma(\phi \to \phi\gamma)$	0.028 MeV	0.044 MeV	$(5.7 \pm 1.4) \times 10^{-2}$ MeV
$\Gamma(\eta' \to \rho\gamma)$	0.039 MeV	(0.062 ± 0.004) MeV	(0.059 ± 0.003) MeV
$\mathrm{BR}(D_s^+ \to \eta'\phi^+)$	3.1	(1.2 ± 0.3)	(3.7 ± 1.2)
$\mathrm{BR}(J/\psi \to \omega\eta)$	2.5×10^{-3}	$(1.58 \pm 0.16 - 2.6 \pm 0.3) \times 10^{-3}$	$(1.58 \pm 0.16) \times 10^{-3}$
$\mathrm{BR}(J/\psi \to \omega\eta')$	5.7×10^{-4}	$(8.9 \pm 1.1) \times 10^{-4}$	$(1.67 \pm 0.25) \times 10^{-3}$
$\dfrac{\mathrm{BR}(\mathrm{J}/\psi \to \phi\eta)}{\mathrm{BR}(\mathrm{J}/\psi \to \phi\eta')}$	0.7	1.42	1.97 ± 0.45
$\mathrm{BR}(J/\psi \to \rho^0\eta)$	2.5×10^{-4}	$(2.12 \pm 0.30) \times 10^{-4}$	$(1.93 \pm 0.23) \times 10^{-4}$
$\mathrm{BR}(J/\psi \to \rho^0\eta')$	5.9×10^{-5}	$(9.1 \pm 1.4) \times 10^{-5}$	$(1.05 \pm 0.18) \times 10^{-4}$
$\mathrm{BR}(J/\psi \to \omega\eta'')$	1.7×10^{-4}	$(1.5 \pm 0.2 - 1.4 \pm 0.1) \times 10^{-4}$	
$\mathrm{BR}(J/\psi \to \rho^0\eta'')$	1.7×10^{-5}	$(1.9 \pm 0.2 - 1.7 \pm 0.2) \times 10^{-5}$	
$\mathrm{BR}(J/\psi \to \phi\eta'')$	8.1×10^{-5}	$(8.6 \pm 0.9 - 7.1 \pm 0.8) \times 10^{-5}$	$< 2.5 \times 10^{-5}$
$\mathrm{BR}(J/\psi \to \phi\eta'')$	5.5×10^{-5}	$(6.0 \pm 0.7 - 5.2 \pm 0.6) \times 10^{-5}$	

This result shows that these states are approximately in an ideal flavor mixing combination. The $\eta(1295)$ is formed by 99.8% of light quarks and the $\eta(1490)$ contains this same percentage of $s\bar{s}$. This result is in agreement with the one given in [18].

4 Conclusions

In this paper we present an extension of a previous η-η' mixing formalism. A glueball was included, in addition to the octet and singlet, as basis for the η-η'-η'' mixing scheme. The glueball mass has been chosen to best fit of the branching ratios and the radiative decay widths involving the η, η' and η'' mesons. Our results are in reasonable agreement with the experimental data and are not very different form those Kitamura et al. [18] and of Genovesi et al. [20]. Nevertheless, we have not assumed a priori any particular value neither for ratio m_s/m_u nor for the coefficients of the eigenvectors. In the extension for the excited states $\eta(1295)$ and $\eta(1490)$ we used the ratio m_s/m_u before determined in the η-η'-η'' mixing. We have taken, as in [5], flavor-dependent binding energies and annihilation amplitudes, however our results indicates 99.8% of $s\bar{s}$ in $\eta(1490)$ in striking agreement with the results in [18].

We emphasize that the distinction of our approach from some previous works consists in the parametrization of the mass matrix. In some works [13,20] the parameters of the mixing schemes are the Euler angles which appear in the coefficients of the physical states in the $q\bar{q}$-$s\bar{s}$-glueball basis. The parameters are fixed by the branching ratios and decay widths calculated with these coefficients. In our approach, the only parameter fixed by the eigenvectors is the mass of the glueball M_G.

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